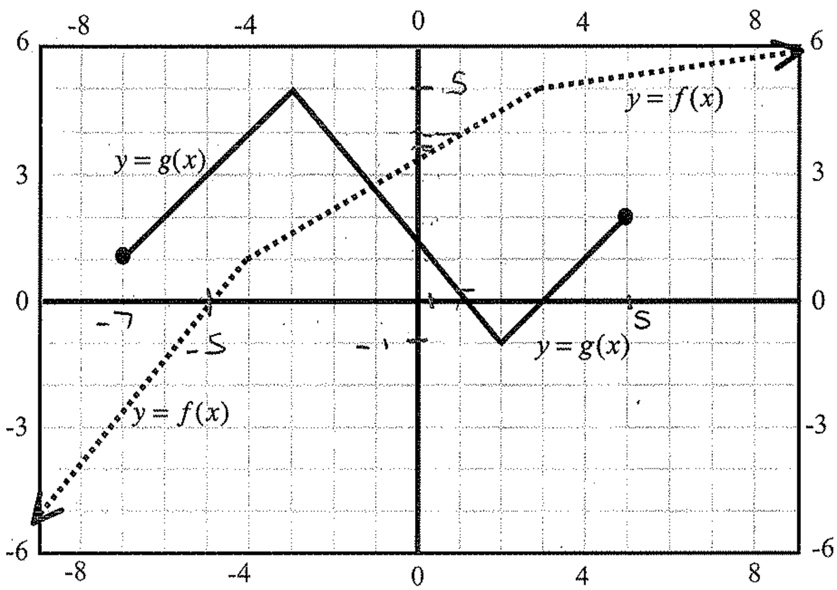
1. [3 pts each] The graphs of the functions f and g are in the sketch shown below. The graph of f is dashed so that you can easily discern which is which. Use the sketch to answer the following questions. Estimate if necessary.



(a) Evaluate $(gf)(0)$ $= g(0)f(0) \approx (3.5)(1.5) = 5.25$

(b) Solve the inequality $f(x) > 0$. $(-5, \infty)$ or $x > -5$

(c) Give the domain of g. Use interval notation. $[-7, 5]$

(d) Give the range of g. Use interval notation. $[-1, 5]$

(e) Evaluate $(f \circ g)(1)$. $= f(g(1)) \approx f(.2) \approx 3.8$

(f) Evaluate $f^{-1}(4)$ $= 1$

For what x is $f(x) = 4$.

2. [2 pts each] Give <u>exact values</u> of each of the following. If an item is not defined, write "undefined."

$\sec 45°$ $= \sqrt{2}$

$\sec^{-1} 2$ $= \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$

$\log_5 \frac{1}{\sqrt{5}}$ $= -\frac{1}{2}$

$\sin^{-1} 2$ undefined

$\ln e^{\pi}$ $= \pi$

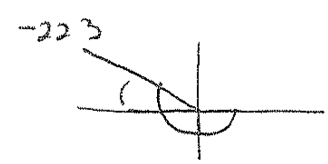
$W(\pi) = (\cos\pi, \sin\pi) = (-1, 0)$

3. [3 pts each] Fill in the missing blank in each of the following.

$(\sin x + \cos x)^2 = 1$ for every real number x is false (true or false)

If x is a positive real number, then $\log_5 x$ is the number t for which $5^t = x$

In solving the equation $\cos x = -\frac{1}{2}$, $\cos^{-1}(1/2)$ is the reference angle of all of the solutions.



The reference angle for $-223°$ is $43°$.

$2\ln x - 3\ln y$ can be expressed as a single logarithm as $\ln\left(\frac{x^2}{y^3}\right)$.

4. [8 pts] Find the domain of the function $g(x)=\sqrt{\dfrac{x-3}{(x-7)(x+4)}}$. Put your answer in interval form.

Require $\dfrac{x-3}{(x-7)(x+4)} \geq 0$

neg under -4 pos 3 0 neg. under 7 pos

Domain $=$ $(-4,3] \cup (7,\infty)$

5. [10 pts] Let $a=\ln x, b=\ln(x+3)$, and $c=\ln(x-2)$. Express $4+\ln\left(\dfrac{x\sqrt{x-2}}{e^3(x+3)}\right)$ in terms of a, b, and c only. (The final expression should not involve e.)

$$= 4 + \ln \frac{x(x-2)^{1/2}}{e^3(x+3)}$$

$$= 4 + \underbrace{\ln x}_{a} + \frac{1}{2}\underbrace{\ln(x-2)}_{c} - 3\underbrace{\ln e}_{1} - \underbrace{\ln(x+3)}_{b}$$

$$= 1 + a + \frac{1}{2}c - b$$

6. [5 pts each] Let. $f(x) = 1 - \ln(8 - 2x)$

(a) Find the x-intercept(s) of f. Exact answers please.

Set $y = 0$

$1 - \ln(8 - 2x) = 0$

$\ln(8 - 2x) = 1$

$8 - 2x = e^1 = e$

$x = \frac{8 - e}{2}$

The x-intercept is $\left(\frac{8-e}{2}, 0\right)$

(b) Find the y-intercept(s) of f. Exact answers please.

Set $x = 0$

$y = 1 - \ln 8$

The y-intercept is $(0, 1 - \ln 8)$

(c) Find the domain of f. Express it in interval form.

Require $8 - 2x > 0$

$8 > 2x$

$4 > x$

Domain $(-\infty, 4)$

7. [5 pts each] Let $f(x) = 8\cos(6x - \pi)$. $= 8\cos 6(x - \frac{\pi}{6})$

(a) Find the amplitude of *f*.

8

(b) Find the period of *f*.

$\frac{2\pi}{6} = \frac{\pi}{3}$

(c) Describe the means by which the graph of *f* is obtained from the graph of $y = 8\cos(6x)$. Your answer should be of the form "Shift $y = 8\cos(6x)$ so many units (right, left, up, down).

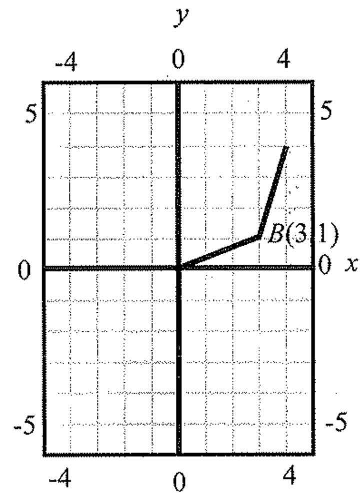
Shift $y = 8\cos 6x$ $\frac{\pi}{6}$ units to the right.

(d) Find the range of the function $g(x) = 2 + 8\cos(6x - \pi)$

the domain of $y = 8\cos(6x - \pi)$ is $[-8, 8]$. For $g(x)$, we shift $y = 8\cos(6x - \pi)$ up 2 units. So the domain of $g(x)$ is

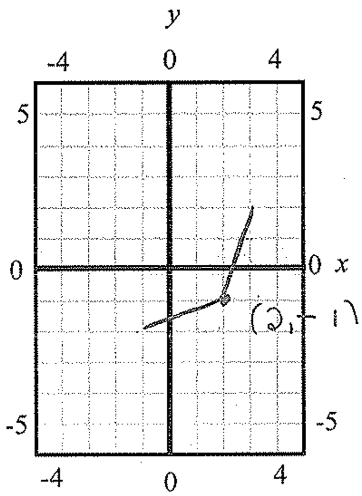
$[-6, 10]$.

8. [5 pts each] The graph of a function h is shown to the right. In each of the parts below, carefully sketch the indicated function. <u>In each part, label the coordinates of the point that corresponds to the "corner point" B in the graph of h.</u>

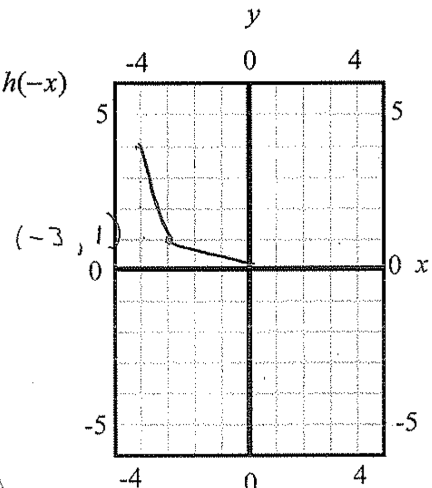


(a) $y = h(x+1) - 2$

Shift 1 left and 2 down



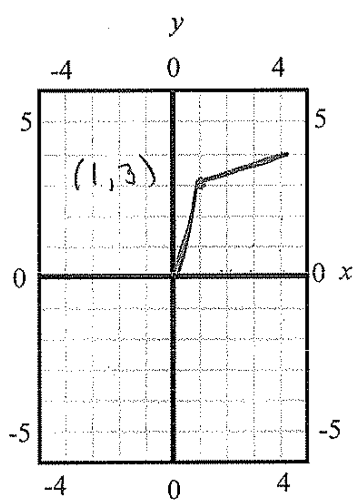
(b) $y = h(-x)$



Reflect through the y-axis

(c) $y = h^{-1}(x)$

Reflect through y = x



9. [9 pts each] Find all solutions of each of the following equations. Your solutions must be exact, and a calculator solution with no work or an approximation is not adequate.

(a) $5+3^{2x-1}=16$

$$3^{2x-1} = 11$$

$$\ln 3^{2x-1} = \ln 11$$

$$(2x-1)\ln 3 = \ln 11$$

$$2x-1 = \frac{\ln 11}{\ln 3}$$

$$2x = 1 + \frac{\ln 11}{\ln 3}$$

$$x = \frac{1+\frac{\ln 11}{\ln 3}}{2}$$

$$= \frac{\ln 3 + \ln 11}{2\ln 3}$$

$$= \frac{\ln 33}{\ln 9}$$

(b) $\sin^2 x+3=3\cos x$

$$1-\cos^2 x + 3 = 3\cos x$$

$$0 = \cos^2 x + 3\cos x - 4$$

$$0 = (\cos x + 4)(\cos x - 1)$$

$\cancel{\cos x = -4}$ or $\cos x = 1$

none here

$x = n\pi$ (n any integer)

(c) $4=3+2\ln(x+1)$

$$1 = 2\ln(x+1)$$

$$\tfrac{1}{2} = \ln(x+1)$$

$$e^{1/2} = x+1$$

$$x = e^{1/2} - 1$$

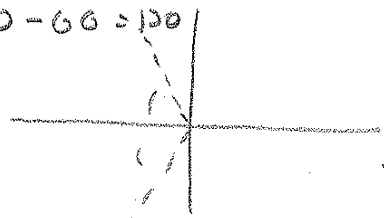
10. [10 pts] Find all solutions of the equation below that are in the interval $[0°, 360°)$. Degree measure is being used here, and exact solutions are required.

$$2\cos(3x)+1=0$$

$$\cos 3x = -\frac{1}{2}$$

$$\cos^{-1} \frac{1}{2} = 60°$$

$180 - 60 = 120$



$180 + 60 = 240$

$$3x = 120 + 360n$$

or

$$3x = 240 + 360n$$

$$x = 40 + 120n$$

or

$$x = 80 + 120n$$

The soln's in $[0°, 360°)$ are

$$x = 40°,\ 160°,\ 280°,\ 80°,\ 200°,\ 320°.$$

11. [10 pts] Let $g(x) = x^2 - 2x$. Evaluate and simplify the following quotient:

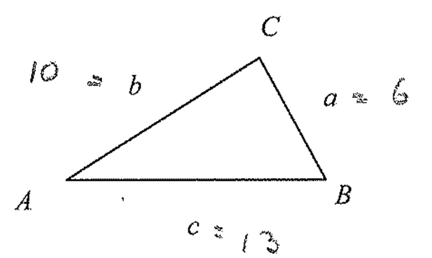
$$\frac{g(t+h)-g(t)}{h} = \frac{[(t+h)^2 - 2(t+h)] - [t^2 - 2t]}{h}$$

$$= \frac{\cancel{t^2} + 2th + h^2 - \cancel{2t} - 2h - \cancel{t^2} + \cancel{2t}}{h}$$

$$= \frac{\cancel{h}\,[2t + h - 2]}{\cancel{h}}$$

$$= 2t + h - 2$$

12. [9 pts] You are given that the sides of a triangle are $a = 6, b = 10, c = 13$ where the angles and sides of the triangle follow the convention shown. Find the angle C to the nearest hundredth of a degree.



By the Law of Cosines

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{6^2 + 10^2 - 13^2}{2(6)(10)}$$

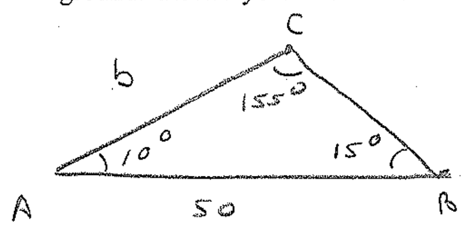
$$= -\frac{33}{120}$$

$$C = \cos^{-1}(-33/120) \approx 106.0^\circ$$

Alzeda

13. [9, 5 pts] An airplane is flying directly between the cities of Alzeda and Busby which are exactly 50 miles apart. An observer in spots the plane, and her angle of elevation to the plane is 10°. At the very same time, an observer in Busby has an angle of elevation of 15° to the plane.

(a) Find the distance of the plane to Alzeda. (Note: This is not the distance along the ground. Round your answer to 2 decimal places.)



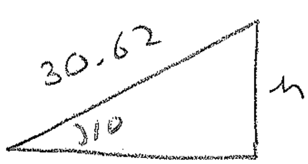
$$C = 180 - 15 - 10 = 155^\circ$$

By the Law of Sines

$$\frac{b}{\sin 15} = \frac{50}{\sin 155}$$

$$b = \frac{50 \sin 15}{\sin 155} \approx 30.62 \text{ mi.}$$

(b) Find the plane's elevation.



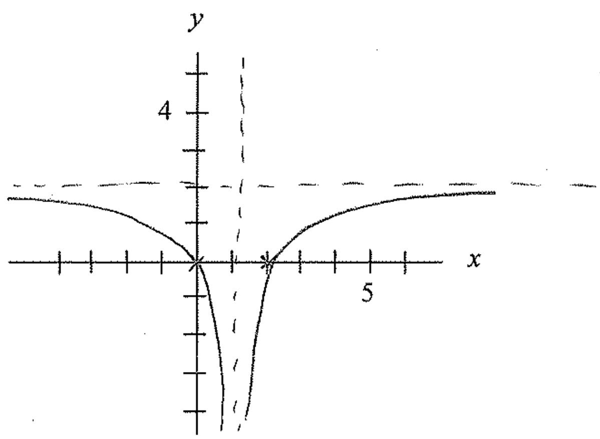
$$\frac{h}{30.62} \approx \sin 10$$

$$h \approx 30.62 \sin 10 \approx 5.32 \text{ mi.}$$

14. [6, 8 pts] (a) A rational function *f* has *x*-intercepts $(0,0)$ and $(2,0)$ and satisfies the following:

as $x \to \infty$, $f(x) \to 2$, and as $x \to -\infty$, $f(x) \to 2$
as $x \to 1^-$,, $f(x) \to -\infty$, and as $x \to 1^+$, $f(x) \to -\infty$

Sketch the graph of f on the axes below. Be sure to also graph any asymptotes of *f* with dotted lines.



(b) Find a valid equation for $f(x)$.

$$f(x) = \frac{2x(x-2)}{(x-1)^2}$$

15. [4, 8, 3 pts resp.] Let $P(x) = 5x^3 - 9x^2 + 8x + 2$

(a) Write the candidates for the rational zeros of $P(x)$ according to the Rational Zero Theorem.

The factors of 2 are 1 & 2, and the factors of 5 are 1 & 5.

The candidates are

$\pm 1, \pm 2, \pm \frac{1}{5}, \pm \frac{2}{5}$

(b) Find all zeros of $P(x)$. Exact answers are required, and complete work must be shown. Solution by advanced calculator is not adequate.

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 8 & 2 \\ & & -1 & 2 & -2 \\ \hline & 5 & -10 & 10 & 0 \end{array}$$

$P(-\frac{1}{5}) = 0$

The complementary factor is $5x^2 - 10x + 10$

$= 5(x^2 - 2x + 2)$, Its roots are

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2} = 1 \pm i$$

The zeros of $P(x)$ are

$x = -\frac{1}{5}, 1+i, 1-i$

(c) Factor $P(x)$ completely into linear factors.

$P(x) = 3(x - (-\frac{1}{5}))(x - (1+i))(x - (1-i))$

16. [10 pts each] Prove each of the following equation is an identities. Be aware that your proof must be logically correct. You are also expected to write the justification for each step (name of the identity used or "algebra.").

(a) $\dfrac{\sin(x-y)}{\sin x \sin y} = \cot y - \cot x$

$LHS = \dfrac{\sin(x-y)}{\sin x \cdot \sin y}$

$= \dfrac{\sin x \cos y - \cos x \sin y}{\sin x \sin y}$ Sum-Diff. Id

$= \dfrac{\cancel{\sin x} \cos y}{\cancel{\sin x} \sin y} - \dfrac{\cos x \cancel{\sin y}}{\sin x \cancel{\sin y}}$ Algebra

$= \dfrac{\cos y}{\sin y} - \dfrac{\cos x}{\sin x}$ Algebra

$= \cot y - \cot x$ Sine/Cosine Id's

$= RHS$

(b) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

$LHS = \cos 3\theta$

$= \cos(2\theta + \theta)$ Algebra

$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ Sum-Diff. Id

$= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ Double < Id

$= \cos^3\theta - 3\sin^2\theta\cos\theta$ Alg

$= \cos^3\theta - 3(1-\cos^2\theta)\cos\theta$ Pythagorean Id

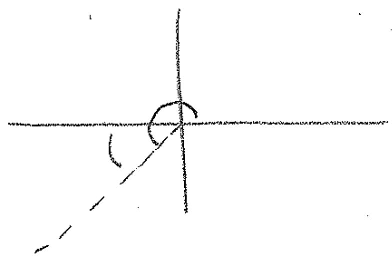
$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$ Alg.

$= 4\cos^3\theta - 3\cos\theta$ Alg.

17. [8, 10 pts] Give an exact value for each of the following:

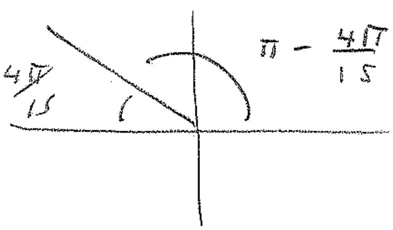
(a) $\cos^{-1}(\cos(19\pi/15))$ — Seek θ in $[0, \pi]$ so that

$$\cos\theta = \cos\frac{19\pi}{15}$$



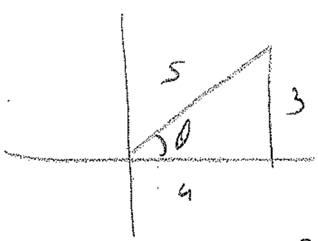
$\frac{19\pi}{15}$ ref $= \frac{19\pi}{15} - \pi = \frac{4\pi}{15}$

$$\cos^{-1}\left(\cos\frac{19\pi}{15}\right) = \frac{11\pi}{15}$$



$\pi - \frac{4\pi}{15} = \frac{11\pi}{15}$

(b) $\cos\left[\sin^{-1}(3/5) - \pi/3\right]$ $= \cos\left(\sin^{-1}\frac{3}{5}\right)\cos\frac{\pi}{3} + \sin\left(\sin^{-1}\frac{3}{5}\right)\sin\frac{\pi}{3}$



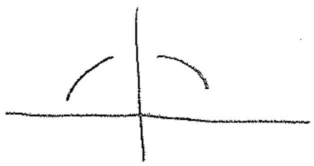
$= \frac{4}{5}\cdot\frac{1}{2} + \frac{3}{5}\cdot\frac{\sqrt{3}}{2}$

$\sin\left(\sin^{-1}\frac{3}{5}\right) = \frac{3}{5}$

$\cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}$

$$= \frac{4 + 3\sqrt{3}}{10}$$

Bonus [5 pts]: Explain why the following statement is true. "The sine function does not distinguish between acute and obtuse angles, but the cosine function does distinguish between acute and obtuse angles."



In Quadrant 1 and in Quadrant 2, the sine function is positive. So the sine of acute and obtuse angles is positive.

In Q1, cosine is positive, and in Q2 cosine is negative. So cosine of an acute angle is positive whereas the cosine of an obtuse angle is negative.

Bonus [3 pts]: State what is used in deriving the Law of Sines. You do not need to derive the Law of Sines. Just give the main item used in its derivation.

Area of a Triangle.

matched problem

Bonus: [2 pts] In the text book, every example is followed by a ______________. (Fill in the blank what the text calls these items.)